

Notations :

On notera la concentration: $n(M, t)$ en particule $\cdot m^{-3}$

Vecteur densité de courant de particule: $j_{diff}^{\vec{}}$ en particules $\cdot s^{-1} \cdot m^{-2}$

$$\text{debit de particule: } \Phi_{diff} = \iint_{\text{Section}} j_{diff}^{\vec{}} \cdot d\vec{S}$$

Loi de Fick :

$$j_{diff}^{\vec{}} = -D \cdot \vec{grad}(n(M, t)) \text{ avec } D \text{ en } m^2 \cdot s^{-1}$$

Equation de diffusion (Cas 1D) :

$$\begin{aligned} \delta^2 N &= j_{diff}^{\vec{}}(x, t) \cdot d\vec{S} \cdot dt - j_{diff}^{\vec{}}(x + dx, t) \cdot d\vec{S} \\ &= -\frac{\partial j_{diff}^{\vec{}}}{\partial x}(x, t) \cdot dx \cdot d\vec{S} \cdot dt \end{aligned}$$

$$\partial^2 N_{prod} = v_{prod}(x, t) \cdot S \cdot dx \cdot dt$$

$$\partial^2 N_{abs} = v_{abs}(x, t) \cdot S \cdot dx \cdot dt$$

d'autre part on a aussi:

$$\begin{aligned} d\delta N &= n(x, t + dt) \cdot dx \cdot d\vec{S} - n(x, t) \cdot dx \cdot d\vec{S} \\ &= \frac{\partial n}{\partial t}(x, t) \cdot dt \cdot dx \cdot d\vec{S} \end{aligned}$$

et comme:

$$\delta^2 N + \partial^2 N_{prod} - \partial^2 N_{abs} = d\delta N$$

$$-\frac{\partial j_{diff}^{\vec{}}}{\partial x}(x, t) + v_{prod}(x, t) - v_{abs}(x, t) = \frac{\partial n}{\partial t}(x, t)$$

$$D \frac{\partial^2 n}{\partial x^2}(x, t) + v_{prod}(x, t) - v_{abs}(x, t) = \frac{\partial n}{\partial t}(x, t)$$

Résistance de diffusion :

en régime permanent et sans production ni absorption: $\frac{\partial n}{\partial t}(x, t) = 0$

$$\frac{\partial n}{\partial x}(x, t) = A = -\frac{j_{diff}}{D}; n(x_2) - n(x_1) = A(x_2 - x_1)$$

$$S(n(x_2) - n(x_1)) = S \cdot -\frac{j_{diff}}{D} \cdot (x_2 - x_1) = -\frac{\Phi_{diff}}{D} \cdot l$$

$$R_{diff} = \frac{n(x_1) - n(x_2)}{\Phi_{diff}} = \frac{l}{D \cdot S}$$

Cas de la 3D :

$$\text{Forme locale : } -\text{div}\vec{j}_{diff}(M, t) \cdot d\tau + v_{prod} \cdot d\tau - v_{abs} \cdot d\tau = \frac{\partial n}{\partial t}(M, t) \cdot d\tau$$

$$-\text{div}\vec{j}_{diff}(M, t) + v_{prod} - v_{abs} = \frac{\partial n}{\partial t}(M, t)$$

$$D \cdot \Delta n(M, t) + v_{prod} - v_{abs} = \frac{\partial n}{\partial t}(M, t)$$

$$\text{Forme intégral : } \Phi(t) + v_{prod} - v_{abs} = \frac{dN_{int}(t)}{dt}$$